However, if you removed all the fractions that included the digit 7 (or, in fact, any specific digit) the total would, in fact, settle to a finite answer. The ways of infinity are strange and wonderful, although proving these results requires fairly advanced mathematics.

11 It might be an interesting experiment to see whether an irregular pentagon (with a relatively small edge for the *do homework* option) might give a less predictable (and more satisfactory) answer.

12 A well-known exercise in a similar vein is *Four-Fours*: to make as many positive integers using exactly four 4s and standard (or even a few non-standard) operations. For example:

$$7 = 4 + 4 - 4 \div 4 \qquad 13 = 44 \div 4 + \sqrt{4} \qquad 22 = 4! \div 4 + 4 \times 4$$

Pupils could be encouraged to find inventive ways of forming calculations involving just the number 2, using as few as possible of them on each occasion.

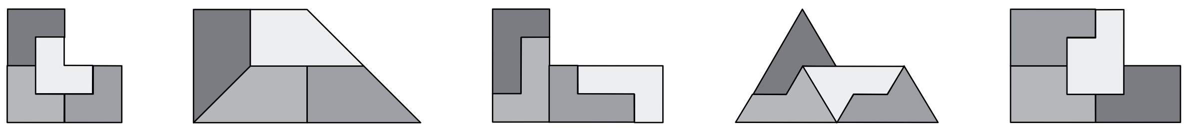
14 Each Gary Baldi biscuit has a mass of 10 g. What mass of biscuits will Marcia get through in a year? The monthly consumption of biscuits in the UK is 34 000 tonnes (http://www.bbc.co.uk/news/uk-england-cumbria-35696027).

15 In a test of 24 questions, what is the *range* of pupils' possible scores. Are some scores within that range impossible, and if so which ones?

16 It is also possible to get another type of hexagon and a *heptagon* as shown below.

How many different numbers of sides could you get from rearranging a trapezium cut into three with two parallel and equally spaced cuts? Or more cuts?

There are certain shapes, known as *Rep-tiles*, which can be divided into smaller copies of the original shapes. A few of the more (or less) obvious are shown below, but pupils may like to find their own.



17 If Jane and Kevin start with the same number of stamps as in the question, is it possible that Kevin could find himself with 3 times as many stamps as Janet, or Janet with 11 times as many as Kevin?

18 The flower beds each have the same area of 12 m^2, but very different lengths for their perimeter. Why is it that the beds with larger perimeters are the ones whose sides are particularly unequal? What shape, maybe not even a rectangle, might you make a flower bed with an area of 12 m^2 so that its perimeter is as small as possible?

20 Could the situation described in the question ever happen in two months in the same calendar year?

25 Another way, albeit not so obvious a way, which offers a little more insight into the problem is to appreciate that 6666 … 666 is $\frac{2}{3}$ of 9999 … 999, which is 1 less than a power of 10. If the original number with 6s has n digits, then it is $\frac{2}{3}$ of $(10^n - 1)$. Multiplying by 6 will give $\frac{2}{3} \times (10^n - 1) \times 6$, which is simply $4 \times (10^n - 1) = 4 \times 10^n - 4 = 4000 \ldots 000 - 4 = 3999 \ldots 996$, a number with $n + 1$ digits, of which $n - 1$ are 9s.

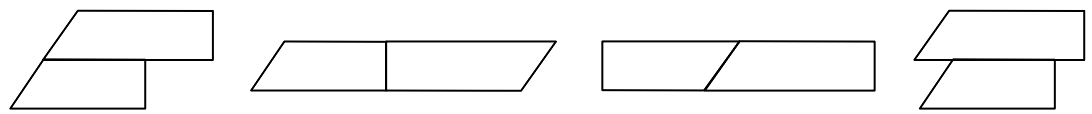
Primary Mathematics Challenge – November 2016

Answers and Notes

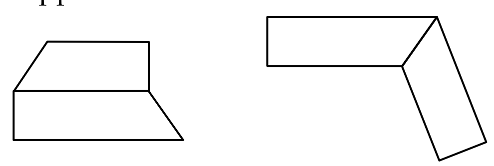
These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 A ($2 \times 0 \times 1 \times 6 = 0$) P2 C ($66 \div 6 = 11$)

1	A	1904	The Olympics in (2016 – 112) were held in St Louis, Missouri in the USA.
2	E	5 for 1	A moment's thought should show that option E is the best deal.
3	E	5 p.m.	Seven hours after 10.00 is 17.00, which is 5 p.m.
4	B	36	One can see that $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = (1 + 8) + (2 + 7) + (3 + 6) + (4 + 5) = 9 \times 4$.
5	C	78	The number of wheels is $3 \times 1 + 14 \times 2 + 9 \times 3 + 5 \times 4 = 78$.
6	A	Sunday	Three weeks, or 21 days, after the first party will also be a Monday, so 20 days after will be a Sunday.
7	C	3 mph	At the same speed Wanda could walk three times as far in 60 minutes.
8	D	£10	One *Ranchie* and one *Winchie* cost 56p + 44p = £1, so 10 of each cost £10.
9	D	35 cm	After cutting off 2×30 cm, he has 140 cm. Dividing by 4, each piece is 35 cm long.
10	D	$1\frac{1}{2}$ and 2	Ten sixths added together is $1\frac{4}{6}$ – this is more than $1\frac{1}{2}$ and less than 2.
11	A	13	Each of the five options is equally likely, so one would expect each $65 \div 5 = 13$ times.
12	E	$2^2 - 2$	$2 \times 2^3 = 2 \times 8 = 16$, $2^2 \times 2^2 = 4 \times 4 = 16$, $2 \times 2 = 4$, $2^3 - 2^2 = 8 - 4 = 4$, but $2^2 - 2 = 2$.
13	E	7	Each of the shapes is based on a 4×4 square and has the equivalent of 4 squares unshaded. Thus they each have an area of $4 \times 4 - 4 = 12$ squares.
14	C	84	Every hour, Marcia eats $4 + 1 + 2 + 3 = 10$ biscuits; there are 8 whole hours between 8.59 a.m. and 5.01 p.m., and she will eat four extra biscuits at 5 p.m.
15	C	24	Suppose that Sofia gets c answers correct and w answers wrong. Her score is $3c - w$. For this to equal zero, it must be that $w = 3c$, that is, for every correct answer she gets three wrong. Since she answered every question, therefore, the total number of questions must be a multiple of 4 – the only such option is 24.
16	B	kite	The rearrangements below show a hexagon, parallelogram, rectangle and octagon.



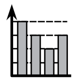
The two shapes have 8 sides altogether. In order to form a kite (or any quadrilaterals other than the parallelogram and square) and thus reduce 8 sides to 4 sides, one would have to join them along sides of equal length. The diagrams below show the two other ways in which this can happen:



and both of these shapes are hexagons.

17 **B** 25 When Jane has half as many as Kevin, she will have one third of the total number of stamps (which stays the same, however many Kevin gives to her). So Jane should end with (123 + 321) ÷ 3 = 444 ÷ 3 = 148. So Jane requires (148 – 123) = 25 more stamps.

18 **D** Doug Holly's bed has a perimeter of 2 × (2 + 6) = 16 m, Basil's is 2 × (4 + 3) = 14 m, Poppy's is 2 × (1 + 12) = 26 m, Doug's is 2 × (0.5 + 24) = 49 m and Rosie's is 2 × (8 + 1.5) = 19 m. Therefore Doug's bed has the greatest perimeter.

19 **A**  The pie chart shows two sectors of 90°, and two other sectors of which one is larger and one smaller. Since the two 90° sectors form half of the chart, so likewise do the other two. The only bar chart that shows this is the first, A.

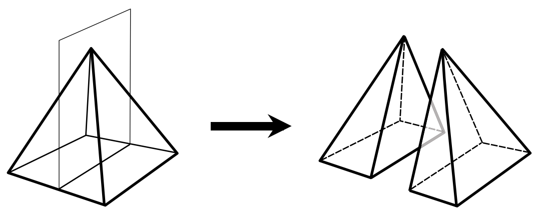
20 **E** Sunday The day numbers that can be square are 1, 4, 9, 16 and 25. As they are separated by a multiple of 7 days, the 4th and 25th days, and the 9th and 16th days will be the pairs that fall on the same day of the week. Since the 25th day is a week and 2 days after the 16th, one may conclude that the 25th (and 4th) are Wednesdays and that the 9th (and 16th) are Mondays. Hence the first day of the month is 3 days before a Wednesday, and so falls on a Sunday.

21 **9.10 a.m.** Between 11.15 a.m. and 5.30 p.m., the machine makes 1000 – 250 = 750 pluffletacks. This takes 6 hours and 15 minutes. At the same rate, it will have taken a third of this time to make 250 pluffletacks, that is 2 hours and 5 minutes. So the machine must have been working for this time before 11.15 a.m. Therefore Mr Pluffle turned the machine on at 9.10 a.m.

22 **2520** The total of the numbers S, O, M and E is 2016 × 4 = 8064. Given that R is 2016 greater than S, the total of R, O, M and E is 8064 + 2016 = 10 080. So the average of M, O, R and E is 10 080 ÷ 4 = 2520.

23 **81** The average number of socks per day is 1001 ÷ 13 = 77. Because the number of socks increases by one each day, and the number of days is odd, Grandpa must knit 77 socks on the middle day, 7 November. Four days later, on 11 November, he will knit 77 + 4 = 81 socks.

24 **8** When the pyramid is cut into two, it forms two smaller pyramids as shown below:



Since each is rectangular-based, like the original, each will have 8 edges.

25 **224** One quickly notices that 66 × 6 = 396, 666 × 6 = 3996, 6666 × 6 = 39 996, and so on. Each product begins with a 3, ends with a 6, and the number of times the digit 9 appears in between is one fewer than the number of 6s in the original number to be multiplied by 6:

$$\underbrace{66 \ldots 66}_{n} \times 6 = 3\underbrace{99 \ldots 99}_{n-1}6$$

For the total to be 2016, all of the nines must add up to 2016 – 3 – 6 = 2007.
Since 2007 ÷ 9 = 223, hence n = 224.

Some notes and possibilities for further problems

P2 The six geese-a-laying feature in the lyrics of the old English Christmas carol, *The Twelve Days of Christmas*, arriving just before the seven swans-a-swimming and after the five gold rings. Pupils may find it surprising to calculate exactly how many gifts are delivered over the 12 days, remembering that each gift is sent not only on the first day it appears, but on all subsequent days. The value of all 364 gifts was calculated in 2015 – it totalled over £120 000.

1 The story of the **1904 St Louis Summer Olympics** is an interesting one, verging on *bizarre*. It was only the third Olympic Games of the modern era, and almost unrecognisable from the event of this past summer. Originally the games were awarded to the US city of Chicago, but the organisers of the *Louisiana Purchase Exposition*, an international cultural and trade fair in St Louis, Missouri, declared that it was preposterous to hold another major world event at the same time, and so decided to bolt on a sporting component to their event. The founder of the modern Olympics, Pierre de Coubertin, eventually gave way and re-awarded the Games to St Louis.
It was not so easy for athletes to travel to North America at the beginning of the twentieth century, and of the 650 athletes who were able to compete fewer than 70 were from outside North America. Not surprisingly the USA came first with 78 gold medals, while Germany struggled into second with only four. In the Marathon, the first to cross the finishing line was the American Fred Lorz, who in hot weather and on dusty roads had suffered exhaustion after 9 miles and was given a lift in his manager's car until that broke down – he then ran the remaining 6 miles to the end and claimed victory. Thomas Hicks, a British runner who turned out to be the real winner, even though he had walked some of the course, had taken rat poison mixed with brandy beforehand to stimulate his nervous system and nearly died after finishing. A Cuban postman, Felix Carbajal, entered right at the last minute, stopped halfway in an orchard for a snack, became violently ill as the apples he had eaten had been rotten, fell asleep, but still continued to finish fourth. And a South-African student, Len Tau, came ninth, though it was thought he would have fared better had he not been chased a mile off course by angry dogs. For more details, see https://en.wikipedia.org/wiki/1904_Summer_Olympics.
The next Olympic Games, held in London in 1908, was hardly better organised or straightforward – to find out why this was is left as an exercise for the reader.

2 Pupils might discuss how they can decide whether, for example, 5 for the price of 4 is better value than 6 for the price of 5; or even which of 3 for the price of 8, or 4 for the price of 11, or 5 for the price of 13 – though frankly none of the last three are hugely enticing deals!

3 As discussed previously (November 2015), allocating time zones is a complex issue, as illustrated by a map of a number of Pacific islands at http://www.worldtimezone.com.

5 Why should *Whizzy Wheelz* stop at 4 wheels per vehicles? There are a number of military vehicles that have 6, 8, 10 or as many as 12 wheels, though these are rather more expensive and exclusive than even unicycles or quad bikes.

10 It is clear that if you were to continue adding sixths forever

$$\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\tfrac{1}{6}+\ldots$$

you would be able to get an answer bigger than any finite number you chose (eventually).

If you replaced the sixths with fractions whose value decreased as their denominator increased, you might expect the result to be different: that is, that

$$\tfrac{1}{1}+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5}+\tfrac{1}{6}+\tfrac{1}{7}+\tfrac{1}{8}+\tfrac{1}{9}+\ldots$$

would "slow down sufficiently" so that it would not get untameably larger. But it doesn't: the value of this too will happily exceed any finite number, however large you chose it.